The Density, and the Twist of the Symbolic

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> Everything is generated from *apeiron* and there its destruction happens. Infinite worlds are generated and they are destructed there again. And he says (Anaximander) why this is *apeiron*. Because only then genesis and decay will never stop.
>
> —Aetius I 3,3<Ps.Plutarch; DK 12 A14.>

> The belief that there is something *apeiron* stems from the idea that only then genesis and decay will never stop, when that from which is taken what is generated is *apeiron*.
>
> —Aristotle, *Physics* 203b 18–20 <DK 12 A 15.>

Abstract

This work does not attempt to map mathematical functions such as y=1/x onto Lacanian schemas as equivalent models. The aim is structural, not analogical. What is at stake is a shared logic: a topology in which relations persist under distortion, stretching, and inversion.

The number line, once cut, already contains this logic. Its asymmetry calls for an operation that cannot be resolved within one dimension. Zero is not a point but a split, marking the limit from which different trajectories emerge depending on how it is approached.

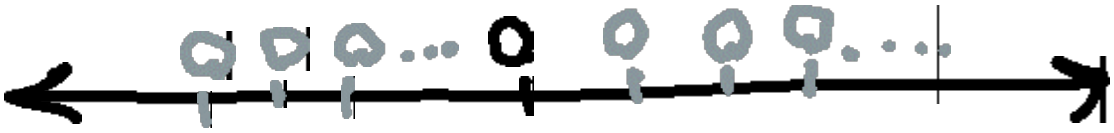
In Lacanian terms, this operation corresponds to the function of fantasy ($◊a). Fantasy does not resolve the impossibility introduced by the cut; it folds the structure around it, allowing a relation to what cannot be addressed directly.

Both the reciprocal structure of y=1/x and Lacan’s Schemas (R and I) reveal this same organization: a hidden Möbius-like logic in which apparent oppositions are sustained by a twist rather than resolved by unity.

> The focus, therefore, is not on formal correspondence, but on the invariance of a structure that holds even when relations are skewed, stretched, or pushed to their limit—where continuity persists through rupture, and where what cannot meet nonetheless remains bound.

Let us begin with an ordinary object: the line of numbers. It appears neutral, transparent, almost trivial. Numbers follow one another at equal intervals. Each unit seems identical to the next. The line looks uniform, measurable, and stable. "*Der Schein trügt!"* (Appearances are deceiving!). This appearance depends on a hidden decision.

Somewhere on this line, a point has been chosen and declared to be zero. Without this decision, the line would have no origin, no orientation, no scale. Every point could be zero, and therefore none would be.

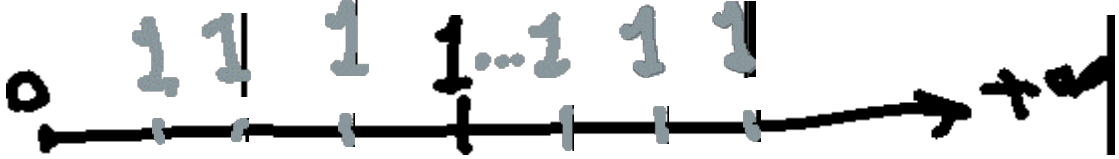


The line would remain what the Greeks called ἄπειρον (*apeiron*): boundless, without articulation.

So the line of numbers is not given; it is instituted/intuited. It begins with a cut. And the cut does not merely *divide*, it emerges the asymmetry as a phenomen (Erscheinung):

Before the cut, any point could divide the line into two equivalent infinities. After the cut, this is no longer possible. The system cannot return to its former symmetry.

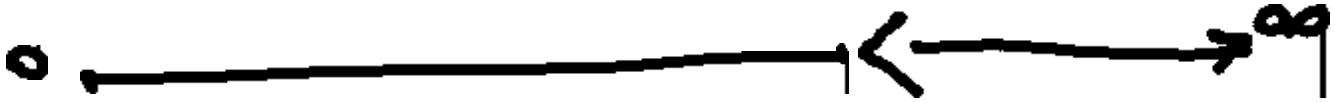
Once zero is installed, the line becomes structured. It is no longer an undifferentiated extension but a field with a boundary (*perion*): one side extends toward infinity where the other retreats from it. But something else happens, something less visible. Distances are still equal in a metric sense, but their structural effect is not.



Z ero and one are instituted differently. Zero emerges through a cut that establishes a boundary, while one emerges through a choice that establishes measure. What appears as a natural order of numbers is in fact the result of these two operations, through which an otherwise indifferent line becomes structured.

Before the cut, the line could be divided at any point into two equivalent parts. Every point could serve as a center, and the division would always produce a symmetry of two infinities. *This was a balance based on equivalence.*

After the cut, this balance is lost. Zero stands for a fundamental loss. The line has become a half-line, and it can no longer be divided into two equal parts in the same way. Any attempt to split it produces a finite segment on one side and an unbounded extension on the other.



The side of infinity always overpowers the finite, no matter how large the latter may be.

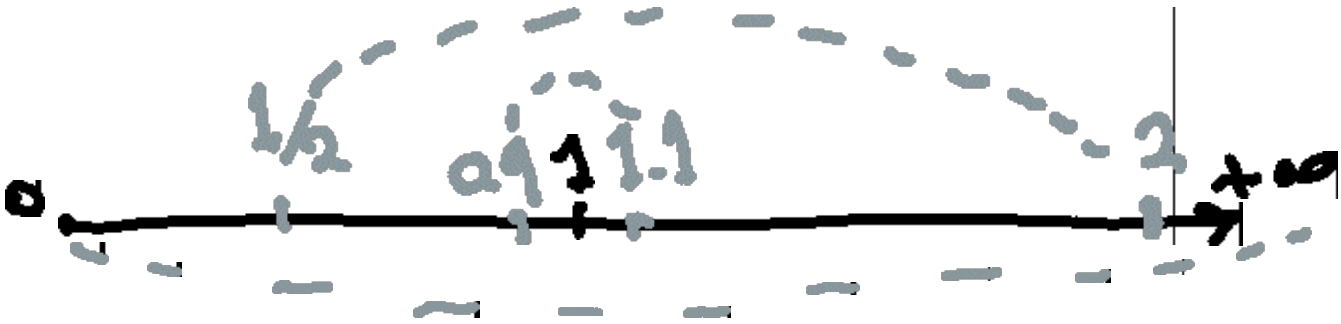
This loss of symmetry calls for a second operation.

If the cut introduced zero as a boundary; establishing lack; then a new principle is required to restore a form of balance. This principle is not another cut, but a *measure*: the *unit*; the institution of *one*.

Zero and one are therefore not symmetric operations. Zero emerges through a *cut* that establishes a *boundary*, while one emerges through a *choice* that establishes measure. What appears as a natural order of numbers is in fact the result of these two distinct operations.

The role of one is decisive. It does not restore the previous symmetry of equivalence; that symmetry is irretrievably lost. Instead, it introduces a new kind of relation: *reciprocity*.

Since the half-line cannot be divided into equal parts, balance can only be achieved by relating one region to another through *transformation*. The point one becomes the pivot of this relation. It allows each position on one side to correspond to a position on the other—not by equality, but by inversion.



Every point in (0,1) is paired with exactly one point in (1,∞) through inversion. Thus, infinity is not elsewhere—it is generated from the neighborhood of zero, where its structure is compressed and made operative.

In this way, what was once a simple equivalence is transformed into a **basculating structure**: a see-saw relation in which contraction on one side corresponds to expansion on the other. The balance is no longer static but dynamic.

It is here that density appears.

Near zero, small variations begin to matter disproportionately. A change from 0.00001 to 0.00000001 is numerically minimal, yet it produces a large transformation elsewhere. What seemed uniform now reveals itself as *elastic*.

This is because the system is not organized by equal steps, but by proportion. What matters is not how much is added, but how much something changes relative to itself. This is widely visible in everyday life: in supermarkets, prices disproportionately begin with “1” rather than “9”; in datasets, leading digits follow the same pattern described by Benford’s law. Such distributions emerge whenever values grow or scale multiplicatively rather than additively.

Near zero, the system becomes dense with distinctions. Small differences carry large effects.

Near zero, the line ceases to measure: it begins to amplify. Density does not refer to the spacing of numbers themselves—they remain evenly distributed—but to the way changes propagate through the structure. Near zero, the system becomes highly sensitive. Small differences produce large effects. The line, which appeared rigid, reveals an underlying flexibility.

Thus, the introduction of zero produces asymmetry, and the introduction of one transforms this asymmetry into a system of reciprocal relations. What was once divided by equivalence is now held together by inversion.

Reciprocal Structures

At this point, we can observe a general feature of such a system. The interval between 0 and 1 can be placed in exact correspondence with the region extending beyond 1. Every point in one region can be paired with a point in the other, such that proximity to zero corresponds to expansion toward infinity. Again, the *near* and the *far* are not opposites, but reciprocally related.

One simple expression of this principle is the relation:

$$x.y=1$$

or:

$$y=1/x$$

But this equation should not be understood as an imported model. It merely makes explicit what is already implicit in the structure of the number line once a cut has been introduced.

What it shows is this: the region (0,1) is not merely finite: it is a zone of compression; within it, the entire structure of the infinite is encoded.

Infinity is not located somewhere else. It is generated through the behavior of the system near zero.

This brings us to a more general idea. In such a structure, certain limits cannot be crossed. Zero cannot be reached from the positive side or the negative side. Each domain approaches it without ever attaining it. Infinity cannot be attained. Yet the system organizes itself around these limits. Relations approach them without ever coinciding with them. These are asymptotic relations. The expression ($y = 1/x$) is one clear instance of this, but the principle is broader: a structure can be governed by elements that are never directly present within it.

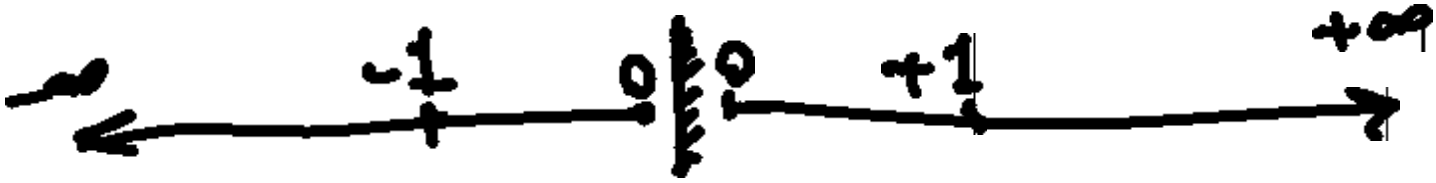
This is not a property of functions alone. It is a property of systems organized around limits. Another consequence follows: the split.

The reciprocal structure does not produce a single continuous field. It produces two domains:

one for positive values

one for negative values

Each domain is complete. Each has its own internal consistency. Yet they cannot be connected. The boundary between them is not a simple gap but a limit of passage.



Each side also organizes itself around a privileged point. In the positive domain, this point is 1; in the negative domain, it is -1. Both are fixed under inversion, each mapping onto itself. But this symmetry is deceptive: -1 is not a second unit, but the reflection of the unit across the cut.

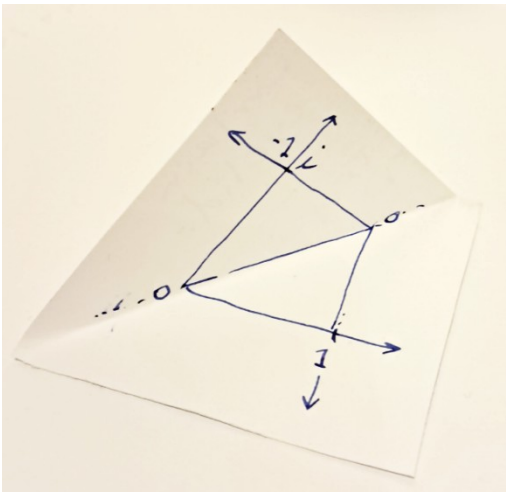
Asymptotic Relations are Topologic

What this all discussions reveal is that topology is not something added later to mathematics. It is already present in the most elementary construction.

The number line, once it is cut, is no longer purely metric. It contains within itself:

- Asymmetry
- Density
- limits that cannot be crossed
- relations preserved under distortion
- …

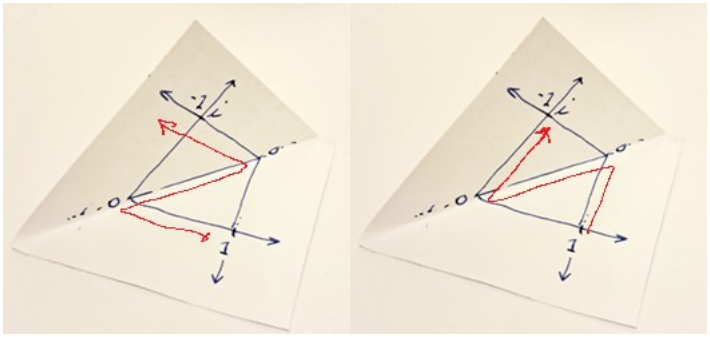
What appears as a rigid sequence of equal intervals conceals a field capable of stretching, compressing, and reorganizing itself. Topology is already there but to perceive it, one must suspend simple scaling and move toward a folded dimension (inspired by the Cathasis vs. Dialation of Don. Kunze):



Zero (now the crease line) compresses infinity; fantasy is the operation that lets the subject live with this compression. The asymmetry introduced by the cut on the number line cannot be resolved within one dimension. The introduction of an orthogonal axis opens a second plane, where what was impossible on the line becomes articulable. The imaginary axis plays a role structurally analogous to the function of φ in the R Schema of Lacan, in that both introduce a dimension that allows the articulation of what cannot be expressed within the original field. The point 0 marks the limit of the real axis, while 1 establishes measure; the vertical direction now introduces an imaginary dimension. The triangle traces the passage between these positions, showing how the structure extends from a linear split into a planar relation. The imaginary axis plays a role structurally analogous to the function of φ, in that both introduce a dimension that allows the articulation of what cannot be expressed within the original field.

The asymmetry introduced by the cut on the number line cannot be resolved within one dimension. The introduction of an orthogonal axis opens a second plane, where what was impossible on the line becomes articulable. The point 0 marks the limit of the real axis, while 1 establishes measure; the vertical direction introduces a dimension that does not extend the line, but transforms its logic.

What remains inarticulable, however, is not simply zero as a point, but the split that zero marks. The crease-line suggested in the figure is not a feature within the space, but the trace of a limit that cannot be represented within it. This is what Lacan designates as the Real: not an element among others, but the impossibility around which the structure is organized.



The different trajectories appear distinct, yet each undergoes the same transformation at the crease. What changes is the path; what remains invariant is the turning. As in the relation −90° = −180° + 90°, different decompositions yield the same orientation. This invariance reveals the nature of the crease: not a boundary, but a twisted seam. In this sense, the fold exhibits a Möbius-like logic—the identification of what appears as two sides as one continuous, non-orientable surface.

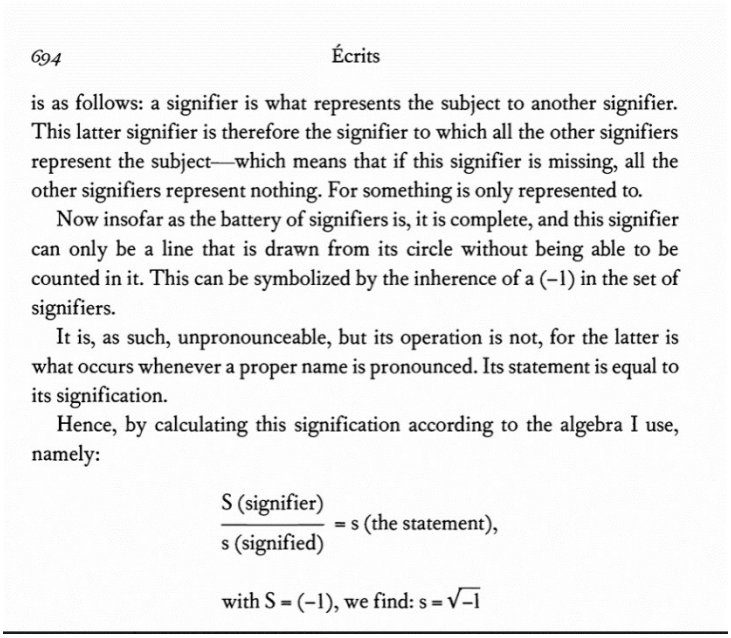
At this point, a further step is required. If zero functions as a limit that compresses infinity into itself, then access to this hidden structure does not come from extending the line, but from a specific operation: the articulation of fantasy.

In *Mr. Know-All* (1938), the mirror stages an encounter that cannot occur directly. The mirror cannot be

crossed as a real act, yet it sustains exchange. What is impossible at the level of passage becomes operative at the level of relation: through crossed gazes, symbols circulate without bodies passing through. The mirror thus functions as a surface of transformation, where separation is preserved but communication persists

In Lacanian terms, fantasy ($◊a) provides the frame through which the subject relates to what cannot be symbolized directly. The diamond of fantasy marks a two-sided relation: the subject is constituted by lack and bound to it as cause of desire. The object a does not complete the subject; it sustains its division. Both are caught in a structure organized around a limit that cannot be crossed: a fold that does not contain them, but conditions their relation. In this sense, the move to an orthogonal dimension can be understood as analogous to the function of fantasy: it does not resolve the impossibility inscribed by the cut, but reorganizes it in a way that makes it operable.

The phallic function (φ) plays a decisive role here. It does not introduce a new content, but a structuring operator that distributes lack across the field. Just as the introduction of an imaginary axis in mathematics allows the articulation of relations that cannot be expressed on the real line alone, the phallic function organizes the subject's relation to lack without eliminating it.



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is as follows: a signifier is what represents the subject to another signifier. This latter signifier is therefore the signifier to which all the other signifiers represent the subject—which means that if this signifier is missing, all the other signifiers represent nothing. For something is only represented to.

Now insofar as the battery of signifiers is, it is complete, and this signifier can only be a line that is drawn from its circle without being able to be counted in it. This can be symbolized by the inherence of a (–1) in the set of signifiers.

It is, as such, unpronounceable, but its operation is not, for the latter is what occurs whenever a proper name is pronounced. Its statement is equal to its signification.

Hence, by calculating this signification according to the algebra I use, namely:

$$\frac{\text{S (signifier)}}{\text{s (signified)}} = \text{s (the statement)},$$

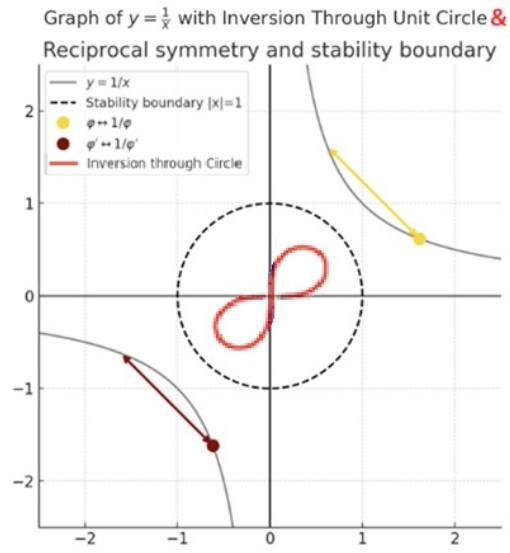
with S = (–1), we find: $s = \sqrt{-1}$

Lacan formalizes the missing signifier as (−1), a term that cannot be counted within the symbolic system yet structures it from within. By assigning this lack to the position of the signifier, he derives $\sqrt{-1}$ as the subject's mode of articulation: an operation that does not resolve absence but displaces it into another dimension. This move anticipates the necessity of extending the structure beyond the line, opening a space in which the subject can be represented only through what is missing.[1]

Thus, what appears in the number line as a hidden infinity compressed at zero becomes, through the logic of fantasy and the operation of the phallic function, a structured field. The passage from line to plane is not a simple extension, but a transformation of the conditions under which relation is possible.

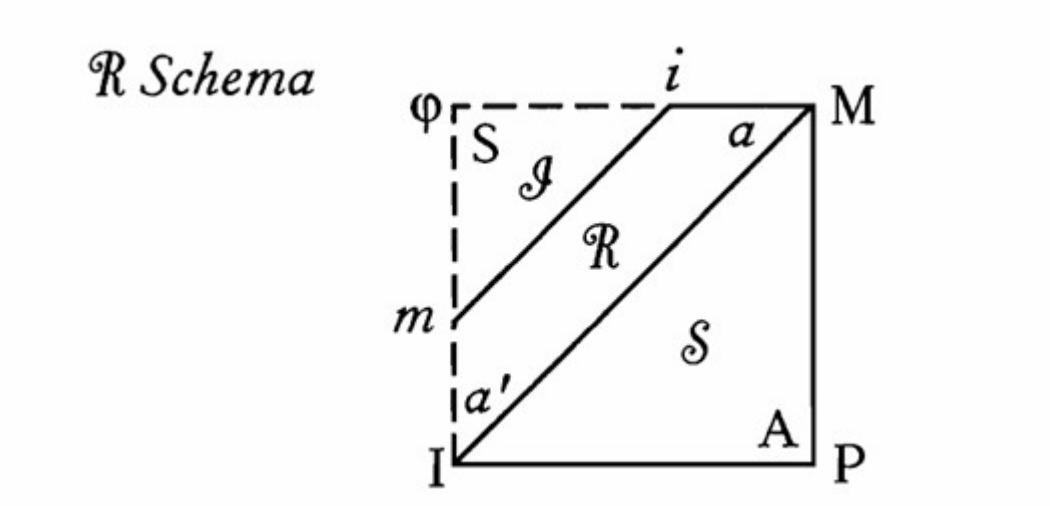
Lacan's Schema R can be approached in this light. But before turning to it, we must recognize that the graph of $y=1/x$ already conceals a Möbius-like logic. The line $x=0$ is not simply a boundary: it is a split, functioning like zero itself: a limit that divides while sustaining a hidden continuity.

[1] Jacques Lacan and Bruce Fink, *Ecrits:* The Subversion of the Subject and the Dialectic of Desire *The First Complete Edition in English* (W.W. Norton & Co, 2006), p.694.

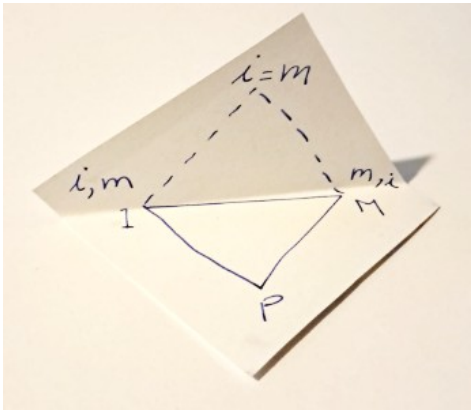


The asymptote hides a twist. Under inversion (via the dotted inversion circle) , the two branches of y=1/x fold into a single trajectory whose trace appears as a figure-eight. This inverted eight is the visible edge of a Möbius-like structure: a surface where inside and outside, near and far, are exchanged without crossing. The apparent rupture of the graph is thus a projection effect: the structure itself is continuous, but non-orientable.

Now, the R schema presents a structured field in which the Real, the Symbolic, and the Imaginary are arranged in a way that allows orientation. Relations are stabilized. The subject can locate itself within a coherent system.



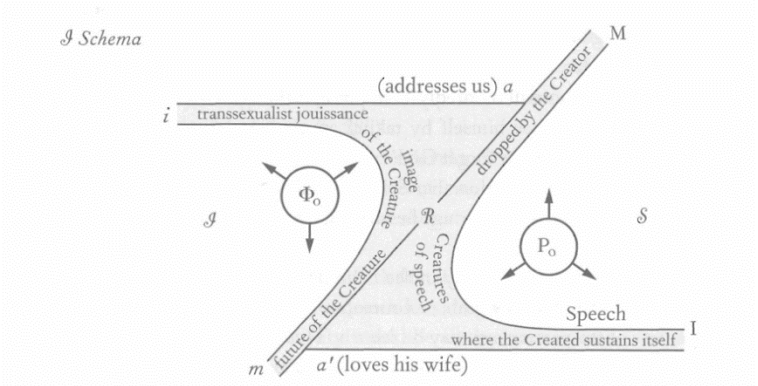
This corresponds to a configuration in which the underlying asymmetry introduced by the cut is still held in check. The field appears consistent because the rupture that founds it does not yet dominate its presentation.



This configuration of Schema R stages a projected unity at the level of fantasy, where i=m appears as if identity were possible. Yet this equality exists only in the dashed, virtual layer. Beneath it, the structure is organized by reversal: the positions of i and m exchange across the field, while the base line

I—M preserves their difference. Anchored by P, the structure resists closure. What appears as unity is supported by a twist that prevents it.

Schema I: The Twist



Schema I introduces a transformation. In Schreber's case, two poles can be distinguished: an intense, invasive jouissance; a preserved relation of love: These are not opposed in a simple sense. They are structurally linked. They form a reciprocal relation in which proximity to one pole corresponds to expansion of the other. The entire experiential field is generated through this relation, not despite it.

Schema I does not destroy the structure. It exposes its condition. The relation persists but no longer as unity. Two trajectories approach without meeting. Yet they are not separate. Their movements correspond. The relation holds because convergence is impossible.

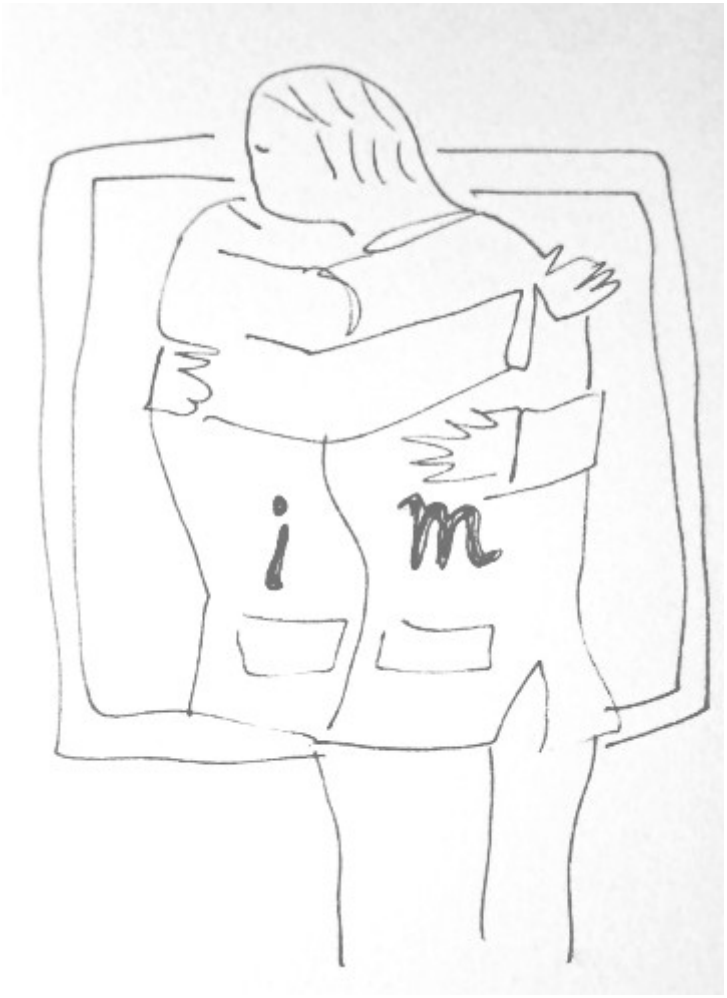
a′: love for the wife

a: transsexual jouissance

They do not reconcile. They determine each other.

Fantasy no longer binds. It exposes the relation it cannot resolve. The result is strict:

a and a′ are asymptotically linked. This is why the wife remains. She is not outside the delusion; she is its asymptote.



i=m

Addendum: The Diamond and the Projective Structure of Fantasy

The relation inscribed by the diamond in $◊a must be freed from any reading that would reduce it to a one-directional causality (this is why Lacan used this diamond sign to designate the two-sidedness and reciprocal relation between $ and a). It does not designate a simple relation in which the object *a* would stand as the cause of the subject's desire. Such a reading leaves intact the illusion that the subject precedes the relation and merely enters into it. On the contrary, the diamond marks a reciprocity that is neither symmetrical nor dialectical, but structural. **The object *a* is not only the cause of desire; it is equally the cause of the subject.** The subject does not stand before this relation: it is produced within it.

One might think here of a mass bound to a gravitational field that bends the space around it as an effect of its being. Yet the inverse is equally true: the gravitational field produces the mass at its very center. Take this only as a metaphor.

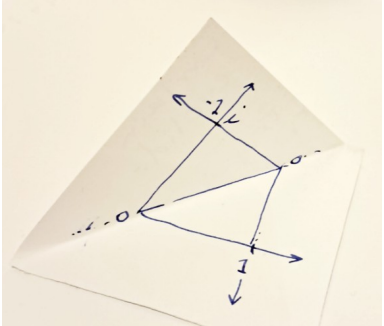
This reciprocity can be rigorously articulated through the logic of asymptotic relations. In Projective geometry, parallel lines do not meet within the finite plane, yet they are not unrelated. Their relation is secured through a point at infinity, where they coincide without ever intersecting in the space in which they are drawn. Their meeting is real, but displaced. It does not occur here. It occurs in a dimension that cannot be localized within the field itself. This clarifies the relation between zero and infinity: they meet not because they are one, but because there is one.

This displacement provides the minimal structure for understanding the relation between the subject and the object. The subject ($) and the object (*a*) do not meet within the representational field. If they did, the structure would collapse either into identity or into contradiction: for instance we cannot embrace our image in the mirror. Instead, their relation is sustained through a point that cannot be inscribed within the field: the place of the Real. It is this point that makes the field possible while remaining inaccessible to it. One may think here of the mercury backing of the mirror: it is precisely because it reflects completely that it cannot itself be seen. The condition of visibility is thus itself withdrawn from what it makes visible. The mercury here in our example belongs to the Real: it is not what we see instead of something else; it is what makes seeing possible while escaping it entirely.

This is what Jacques Lacan isolates when he situates the subject not in the picture, nor in the mirror, but in the interval structured by two parallel lines, this “space of withdrawal” through which fantasy becomes possible. I am actually referring to a precise phrase used by Jacques Lacan in Seminar XIII (The Object of Psychoanalysis), session XIX of May 25, 1966 (p.232):

> …this space of withdrawal (espace de recul) between two parallel lines, thanks to which, always insufficient but always desired, at once do-able and impossible, the phantasy can be summoned to appear in some way in the picture.

These two parallel lines must not be understood as two independent trajectories. They are the visible effect of a split introduced into a single structure. The cut; marked, in the present work, by zero; produces an asymmetry that cannot be resolved within one dimension. The line folds. What was one appears as two.



Let us revisit this image again: What appears as two axes is the unfolding of a single line split at zero. What could not be articulated on the line reappears as

operation in the imaginary plane. The triangle traces these operations: each edge a displacement that preserves the split while reconfiguring its effects. In this sense, the apparent separation becomes a command: *meet!* Not as coincidence within the field, but as a projective relation sustained beyond it. What cannot meet is nonetheless bound; held together in the space of recession where fantasy operates, making the impossible relation function.

Thus, the two lines are not two. They are one, divided by a limit that cannot be crossed. Their apparent parallelism is the effect of this division. Their meeting at infinity is the structural truth of their unity. This is why no angle can be assigned to their intersection. Angle belongs to the metric order of representation. But the relation at stake here is projective: it concerns incidence, not measure. The lines meet without crossing, just as the subject and the object are bound without coinciding.

The formula of asymptotic reciprocity makes this explicit:

$$y=1/x$$

As one term approaches zero, the other diverges toward infinity. The two trajectories are inseparable, yet they never meet within the plane. Their relation is exact, but it is not localizable. Zero must therefore be understood not as a point, but as a split: a limit that both divides and binds. It is the site at which infinity is compressed, and from which the two branches emerge as distinct.

This structure finds its clinical articulation in the case of Judge Schreber, as formalized by Jacques Lacan in Schema I. Two poles organize Schreber's experiential field: transsexual jouissance (a) and love for the wife (a'). These are not opposed contents, but asymptotically related terms. They do not meet, they cannot be reconciled, yet neither can be thought without the other. Their relation is exact, but non-coincident. As one intensifies, the other is displaced. What appears as division is in fact a strict reciprocity governed by a limit that cannot be crossed.

At the same time, Schreber's delusion articulates another pair: creature and creator. Here again, the same structure emerges. God does not stand outside the system as an absolute origin. He is caught within the same relation. Schreber is created by God, yet God depends on Schreber for his completion. The direction of causality is inverted. The creator is sustained by the creature. What appears as transcendence reveals itself as structural dependence.

These two pairs—a and a', creature and creator—are not distinct. They are two expressions of the same relation. And this allows a decisive formulation: the object that causes desire is identical to the object that causes the subject. The subject emerges as an effect of the same structure that it takes as its cause. There is no exteriority between them. Their distinction is maintained only through the impossibility of their direct encounter.

In this sense, the diamond ($\$\lozenge a$) does not connect two already constituted terms. It produces them as reciprocally dependent. It inscribes a relation in which each term exists only through its non-coincidence with the other. This is not a synthesis, nor a reconciliation, but a torsion. A Möbius-like structure in

which inside and outside, cause and effect, subject and object, are not opposed but continuously transformed into one another without ever becoming identical.

Fantasy occupies the place of this torsion. It does not resolve the impossibility introduced by the split; it organizes it. It provides the frame in which the non-meeting of the subject and the object can be sustained as a relation. This is why Lacan designates the picture (Las Meninas) not as representation, but as Vorstellungsrepräsentanz: the representative of representation. What is presented is not the object, but the condition under which representation itself becomes possible.

The meeting of parallel lines at infinity is therefore not a metaphor for fantasy. It is its structural condition. What appears as two is one. What cannot meet is nonetheless bound. What is divided is held together by a point that cannot be placed within the field. This point is not visible. It is the place from which the relation is sustained. And it is from there that the subject can say: “*you do not see me from where I am looking at you.*”